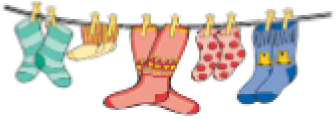
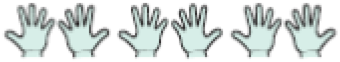
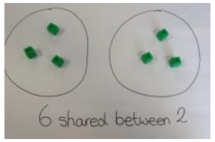
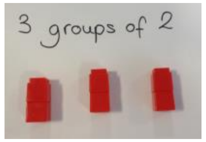
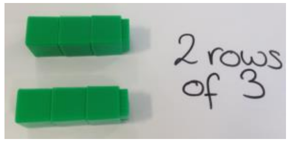
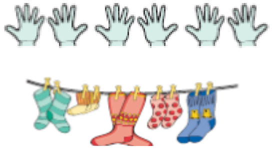
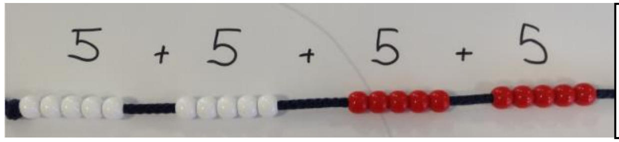
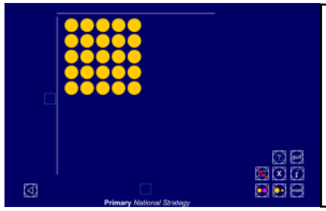
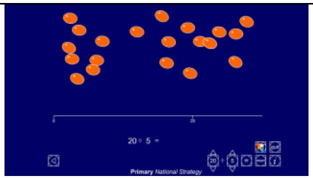
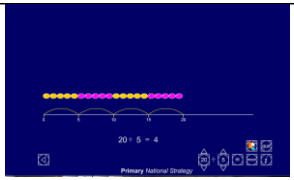
Developing lifelong learners

Russell Hall
Primary School

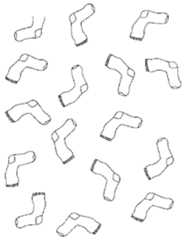
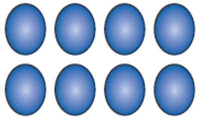
This is the calculation policy for Russell Hall Primary School (created in association with Sharon Day of SharonDayMaths Ltd.), ready for the Autumn term 2015, which lists the progression through division and multiplication. YR, Y1 and Y2 and Y3 are mostly working mentally (which means using concrete resources to build conceptual understanding of the operations and then using growing knowledge of times table facts). The signs and number sentences for division and multiplication are introduced throughout year 2. During year 4 the formal written methods for multiplication and division are introduced as the children are working with numbers which demand this. Long multiplication is introduced in year 5 and long division, should it be required, is introduced in year 6. Mental mathematics runs throughout with the children being trained to study the numbers before they start to decide on the most efficient method for working it out. The words in standard font are taken from the NC programme of study. Reference should be made to this document when planning to ensure that other skills from other domains that feed into calculating are utilised i.e. the domain for Number and Place Value. ***The words in italics are guidance put together by the school to support teachers with the delivery of the policy.***

Year	**division**	**multiplication**
FS	ELG 11: Children solve problems using doubling, halving and sharing.  *Provide opportunities to have experience of recognising where items are organised into equal groups and then putting together equal groups of items in various areas of provision.*    *'How many bags of apples with five in each bag can you make?' 'I can make two bags of five apples.'*	
1	Through grouping and sharing small quantities, pupils should begin to understand multiplication and division; doubling numbers and quantities, and finding simple fractions of objects, numbers and quantities.    	

	Solve one-step problems involving division *(experiencing sharing **and** grouping*), calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. *(No expectation for recording formally at this stage.)* Share these pencils equally between Asif and Ben. How many pencils will each of them get?  How many children can have two squares each from this chocolate bar? 	Solve one-step problems involving multiplication, calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. *(No expectation for recording formally at this stage.)* Count the eggs in this egg box  *It is promoted and then expected that the children will count in twos using two extended fingers to keep track of the count.*
2	Pupils solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts. *(i.e. division is 'grouping' only for calculating and 'sharing' by 2 is only referred to during halving activities)* Pupils solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts. *(ensure that division is 'grouping' for calculating and 'sharing' by 2 is only used during halving activities)*  *Four groups of five is written as 5+5+5+5 which is 'Five, four times' which is eventually written as 5 x4 as the first number in the calculation is the amount we know – the group being multiplied.* *Array ITP:*  *This array shows: 5x5=25; 25=5x5; 25÷5=5; twenty-five in groups of five gives five groups of five.* *half of 14 is 7 (in each half):*  *Grouping ITP:*	

<table>
<tr>
<td></td>
<td colspan="2">Half of 50 is 25 (in each half):     </td>
</tr>
<tr>
<td></td>
<td>Calculate mathematical statements for division within the multiplication tables of 2, 5 and 10, writing them using the division (÷) and equals (=) signs
(How many groups/sets/lots of two do we use to make fourteen?)…
‘Fourteen divided by two is seven’: 14 ÷ 2 = 7
(How many groups/sets/lots of five do we use to make forty-five?)…
‘Forty-five divided by five is nine’: 45 ÷ 5 = 9

This is NOT ‘sharing’ – it is organising the dividend into GROUPS of the divisor.

Using the concept of grouping to support with learning tables facts:
How many pairs of socks are there?

There are other questions that may demand the use of sharing such as ‘halving and halving again’ as with this:</td>
<td>Calculate mathematical statements for multiplication within the multiplication tables 2, 5, and 10, writing them using the multiplication (×) and equals (=) signs
When using an array read it from left to right, so this image is ‘Two, four times’ or ‘2+2+2+2’ or ‘Two times by four’ or ‘Two multiplied by four’:

$2 \times 4 = 8$

‘Two times by seven is fourteen’: 2 x 7 = 14
‘Five multiplied by nine is forty-five’: 5 x 9 = 45

And in contexts:

‘How many two pence pieces do you need to make 20p?’

There are 10 crayons in each box.

How many crayons are there altogether?</td>
</tr>
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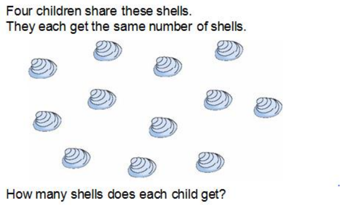
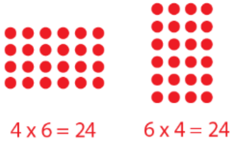
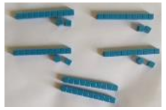
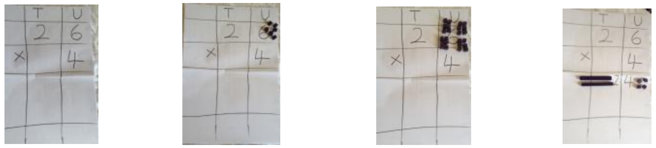
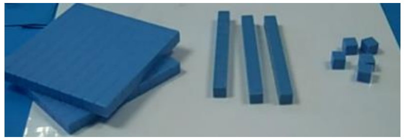
<tr><td></td><td>
Four children share these shells.
They each get the same number of shells.
How many shells does each child get?</td><td></td></tr>
<tr><td>3</td><td colspan="2">Pupils develop use of mental methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the efficient written methods of short multiplication and division (using the times tables of 2, 3, 4, 5, 8 and 10).
<i>Explore arrays with the times tables they know to understand that once I know one fact I know more than one fact (i.e. the first image shows four multiplied by six or 4+4+4+4+4+4 and the second image shows six multiplied four or 6+6+6+6 but they both answer twenty-four):</i>


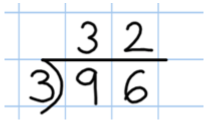
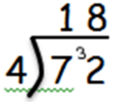
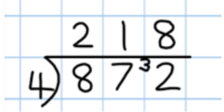
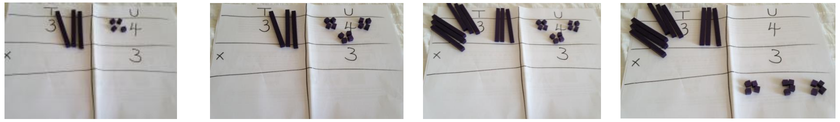
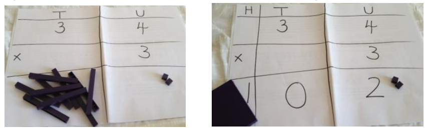
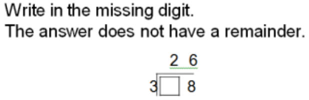
<i>Mental methods will include:</i>
- <i>partitioning amounts in different ways using base 10 resources in response to being equally divisible by the above multiples i.e. ‘Put sixty-eight into different arrangements so that each part of it is equally divisible by four.’</i>
    
<i>The children can begin to record findings after they have created a variety of responses with the concrete resources (manipulatives). Encourage them to work systematically to find all possibilities. i.e. 68=60+8, 68=40+20+8, etc.</i>
- <i>use of a (vertical or horizontal) number line/jottings to record the amount of groups (the divisor) counted up to that gets you to the amount being divided (the dividend)</i></td></tr>
<tr><td></td><td>Divide two-digit numbers by a one-digit number

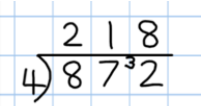
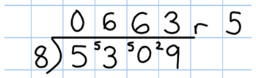
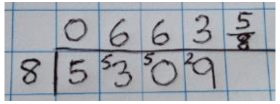
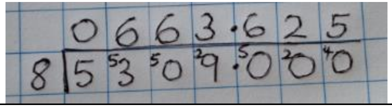
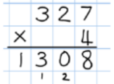
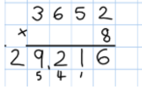
<i>The children are given plenty of practical work and are encouraged to use mental strategies and informal pencil and paper or whiteboard jottings to support, record and to explain their thinking.</i>

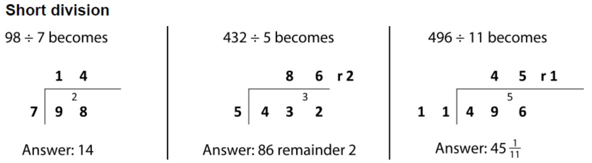
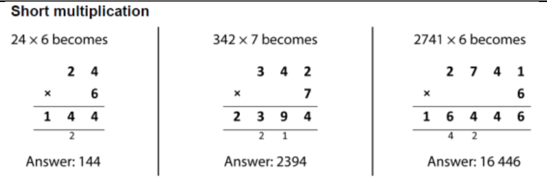
<i>Children are taught to solve division calculations by using</i></td><td>Multiply two-digit numbers by a one-digit number

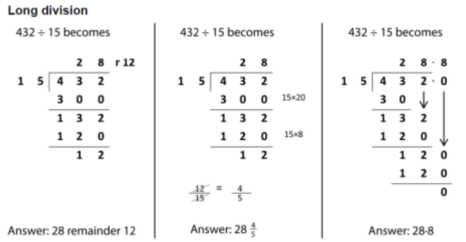
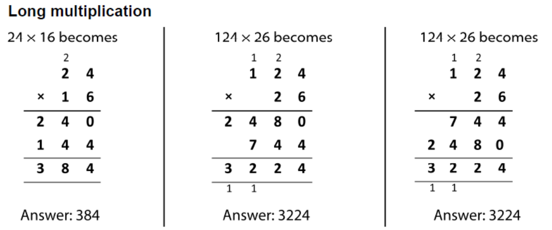
<i>This could start with expanded method and would look like:</i>
<i>26</i>
<i>X 4</i>
<i>24</i> <i>(show with base 10 resources where these products have come from)</i>
<i>80</i></td></tr>
</table>

	multiplication strategies e.g. calculate 18÷3 by counting on in multiples of three or by recalling tables facts this could be modelled by keeping track of the number of groups of three with fingers where each finger 'stands for' 3.. The link between division and counting on in groups of the divisor should be made. This then progresses to dividing larger two-digit numbers i.e. 68÷4 where the number 68 (built with ten sticks and unit cubes is partitioned into 40 and 28, so that both parts are divisible by the divisor).	*104 (show with base 10 resources from above where this total has come from)*  *Build then multiply the unit value, placing it in the answer 'box'.*  *Build then multiply the tens value and place in the answer 'box'.*  *Find the total on the two products by using the addition method.*
Examples easily calculated mentally	*24÷4 (as this can be done mentally using known facts but could be laid out in the short division format to promote familiarity; continue to use arrays to understand the concept)* *'I know that 4x6=24 so then 24÷4=6'*	*5x4 (as this can be done mentally using known facts; continue to use arrays to understand the concept)* *3x10 (as this can be done mentally by using the learned effect of multiplying by ten – NOT 'adding a zero' - the number becomes ten times bigger)*
4	Pupils should practise to become fluent in the efficient written method of short multiplication for multiplying using multi-digit numbers, and short division with exact answers when dividing by a one-digit number (with all 12 times tables facts)  ***Practising partitioning numbers in different ways, in response to investigating in the context of a variety of divisors, supports children with understanding division and multiplication i.e. 'Make the number 235 with base 10 resources. Now move the resources around to make different numbers that are equally divisible by 5.' After exploration the children can begin to record such as: 200+30+5; 100+100+20+15; 100+100+10+10+5; etc. 'Now move the same amount around into multiples of 4. What do you notice?' 100+100+20+12 with 3 left over etc.***	
	Divide two-digit and three-digit numbers by a one-digit number using	Multiply two-digit and three-digit numbers by a one-digit number using

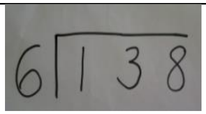
	formal written layout *In order to become familiar with the short division layout begin with numbers that do not have remainders moving to calculating with numbers that require one amount to be 'carried'. Demonstrate for all children without the concrete resources then if there are children who question the validity of the method regarding it linking to place value show this group of children how the method works in relation to place value using base ten resources (see the example at the end of this document):*  *moving to*  *and*  ***See at the end of the document for discussion on short division method as well as details on how base ten resources could be used to help with conceptual understanding***	formal written layout. *The following uses the example of 34 x 3 ('thirty-four multiplied by three'; 'thirty-four, three times'):*  *Build 'top number'; multiply the units then the tens; place the answer*   *for the units and 'carry'; place the answer for the tens and 'carry'.* ***See at the end of the document for the above in more detail, with commentary.***
Other examples	*Calculate 56 ÷ 4* *456÷4* *To ensure children grasp how the layout for short division works ask them to respond to 'rich and sophisticated' questions that demand reasoning, such as:*  Write in the missing digit. The answer does not have a remainder. $\begin{array}{r} 2\;6 \\ 3\overline{)\square\;8} \end{array}$	*Calculate 58 × 6 (where they have to set it out vertically from this)* *To ensure children grasp how the layout for short multiplication works ask them to respond to 'rich ad sophisticated' questions that demand reasoning, such as:* *Write in the missing digit.* 5☐ × 8 456
Examples when a written method may not be	*97 ÷ 100 (this can be done using knowledge of what happens to a number when divided by 10, 100, etc.)* *242 ÷ 2 (this can be done by halving)* *55÷5 (this can be done using known facts)*	*97 x 100 (this can be done using knowledge of what happens to a number when multiplied by 10, 100, etc.)* *33 x 2 (this can be done by doubling)* *3 x 70 (this can be done by combining known facts with multiplying by ten)*

needed		
5	Pupils practise and extend their use of the formal written methods of short multiplication and short division. They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.	
	Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.
examples for written	*Start with amounts where the first digit in the dividend is larger than the divisor:*  *Moving to amounts where the first digit in the dividend is smaller than the divisor:*  *Express answers as fractions (here there is a remainder of five out of the eight being divided into it so the remainder is 'five out of eight'):*  *Express answers as decimals by 'carrying' into the decimal columns:* 	*Conceptual understanding of long multiplication (using manipulatives):* *Short multiplication that requires 'carrying' (multiplying by a single digit number): i.e. 327 x 4 and 3,652 x 8*  
examples for mental	*Any calculations where the dividend is a multiple of the divisor, known facts can be used and 'carrying' does not have to occur i.e. 8,884 ÷ 4* *Write in the missing number:* *3400 ÷ ▯▯= 100* *(use their knowledge of place value)*	*Where the calculation demands moving a number in response to place value i.e. 452 x 1,000* *What is double fifteen point five?* *Write in the missing number:* 8 × ☐ = 400

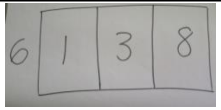
	Divide nought point nine by one hundred. (move the digits two places to the right)	
6	Pupils practise multiplication and division for larger numbers, using the formal written methods of short and long multiplication, and short and long division.	
examples for mental	*Divide thirty-one point five by ten* *Ten times a number is eighty-six. What is the number?* Circle the best estimate of the answer to 72.34 ÷ 8.91 6 7 8 9 10 11	*What is nought point eight multiplied by six?* *What must you multiply nought point seven by to get two point one?* *A bag of four oranges costs thirty seven pence. How much do twelve oranges cost? (understanding how to use knowledge of related multiplication tables i.e. here the 12 times table is three times the four times table so to solve this you just need to multiply 37 by 3.)*
Final outcomes for short formal written methods *(where the number is being divided or multiplied by a single digit number)*	 *The answers to short division, where the dividend cannot be divided equally by the divisor, can be shown as a remainder as well as fraction. Showing the remainder as a fraction is easy as we are just writing the remainder at the top of the fraction (numerator) and the divisor at the bottom of the fraction (denominator). 'We had one left over and we were dividing it by eleven so the remainder is written as one out of eleven'. It is also possible to use the short division method where the divisor is a larger two-digit number thus making the long division method unnecessary – the answer can be written as a decimal. The quotient can be expressed in different ways depending on context of how the context is expecting the answer)*	 *When teaching short multiplication we are working from the right as with addition and subtraction methods shown in columns. To support children with understanding about 'carrying along' where the product is a two-digit numbers use base 10 resources to model the amount that has been made (this builds on the formal method they will have learned for addition). So, in the first example when we calculate that six fours is twenty-four (hopefully using known facts) then we build the twenty-four displaying the 4 units in the Units column and the 2 tens carried over to the tens column. This is then combined with the product of six twenties to make 14 tens which is then recombined as one hundred flat and four ten sticks.*

Final outcomes for long formal written methods *(where the number is being divided or multiplied by a two digit number which is greater than 12)*	 *Long division can be done two ways and both ways have their reasons (and again the quotient can be expressed in different ways depending on context of how the context is expecting the answer):* *1) We think of the number we are dividing (the dividend) as the full place value that they have (see the first two examples below – the second uses jottings to keep track of how many groups of the divisor have been used).* *2) We think of the number we are dividing (the dividend) as if they are separate two-digit numbers.* *Teach one of these versions to your class and any children still struggling with the version you have chosen after a while teach the other version as that may be the one that makes the most sense to them.*	 *Long multiplication can be done two ways and both ways have their reasons:* *1) Starting by multiplying the top number by the unit of the second number down builds on where the children start when they use the method for short multiplication.* *2) Starting by multiplying the top number by the tens of the second number more closely matches what we do with grid method.* *Teach one of these versions to your class and any children still struggling with the version you have chosen after a while teach the other version as that may be the one that makes the most sense to them.*
MORE ABOUT SHORT DIVISION LAYOUT	*For many teachers the short and long division formal method is problematic. It is a useful and efficient way of dividing as it is a quick method however some teachers feel an aversion to it as it can be learned as a trick with no understanding. Ironically it seems to be that children with a poorer understanding of place value pick the method up quicker, as children with good place value will often question how it works rather than just accept it as a 'trick'. There are ways to attempt to link place value and models to the method and below is a series of images to attempt to illustrate these. One way uses bundles and refers to the digits in the dividend as individual digits and the second way uses base ten resources with an attempt to link the method to place value. I advise that you demonstrate the method to the majority of children without concrete resources initially then use the concrete resources in a guided session for children who do not acquire the method readily or who are questioning how it works in response to place value.* ***The digits can be separated should it be required, but it does not have to be drawn in this way****:*	

So for this calculation:



draw a grid with the digits separated:



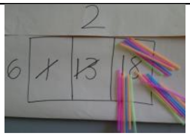
Using bundles of straws (in tens) and single straws begin to work through it a section at a time:
'How many groups of sixes can you get out of 1?' 'None.':

'So move the digit not used across to the next column and then build the new number (which is now thought of as thirteen) 13 with the straws' (one ten and three units):

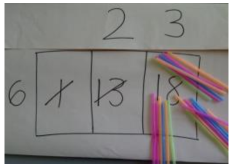
'How many groups of six can you get out of thirteen?' 'Two groups of six with one left over (remaining)':

'Carry the remaining digit over to the next section and then build the new number':

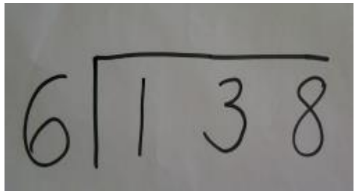
'How many groups of six can you make with eighteen?' 'Three groups of six':



'So the answer to one hundred and thirty-eight divided by six is twenty-three groups of six':

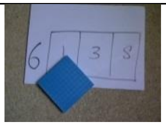
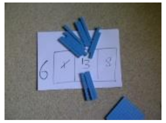
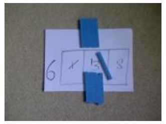
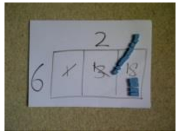
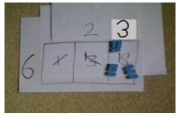
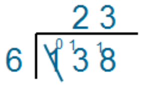


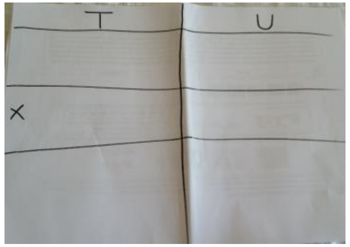
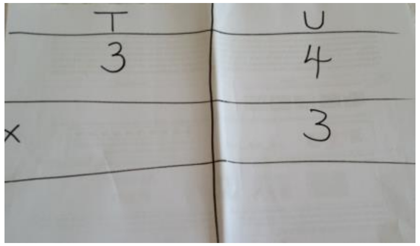
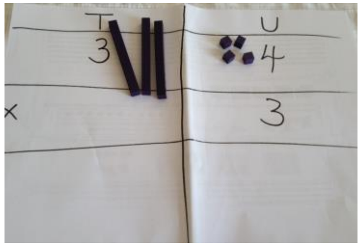
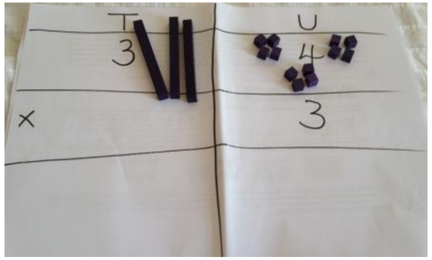
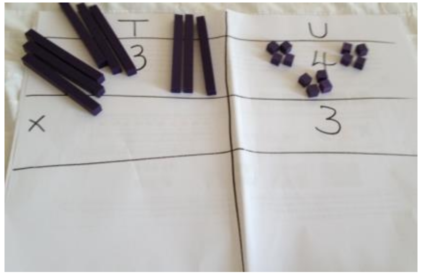
The above could also be demonstrated using 'Base Ten' equipment which takes more account of the place value as follows. Use this for children who need to know how it works. Do not bother with this for children who are able to do the method 'as a trick' and are not questioning how it works unless you feel they would benefit from it. The important principle here is that as you divide in each column you make groups of the divisor multiplied by the value of the column so, if you are dividing 8 ones into groups of 2 then you would describe it as 'Eight ones put into groups of two ones is four groups of two ones.' You wouldn't just say 'Eight divided by two is four'. So if you were in the Tens column and it contained eight tens (80) you would say: 'Eight tens put into groups of two tens are four groups of two tens.' Hopefully the images below will help to clarify how to describe what is happening:

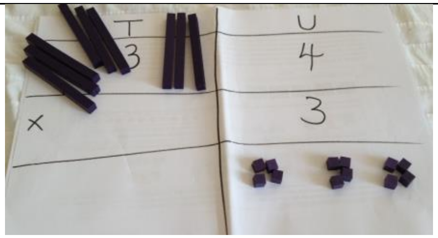


So for this calculation: again draw a grid with the digits separated if you require it:

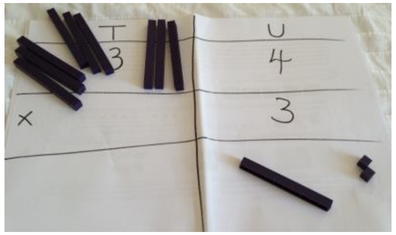
Using base 10 resources begin to work through it a section at a time:
'Can I make a ***group of six 'hundreds'*** *from this model?' 'No because there is only one 'hundred.':*

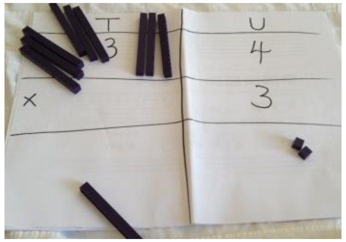
	 *‘So move the digit not used across and then build the new number (which is now thought of as **thirteen tens** because it is in the tens column) with ten sticks’:*  *‘Can I make any **groups of six tens**?’ ‘Yes. Two **groups of six tens** with one ten stick left over (remaining)’*  *‘Carry the remaining digit over to the next section and then build the new number’:*  *‘How many **groups of six ‘ones/units’** can you make with eighteen ones/units?’ ‘Three **groups of six ‘ones/units’**:*  *‘So the answer to one hundred and thirty-eight divided by six is twenty-three **groups of six’**:* 
Multiplication	*This demonstrates how the written method of short multiplication can be taught to children with conceptual understanding related to*

method in more detail	*place value. The following uses the example of 34 x 3 ('thirty-four multiplied by three'; 'thirty-four, three times'):* *Draw a grid labeled with tens and ones and then build the number being multiplied (called the multiplicand) which is usually the larger amount of the two for ease:*    *Multiply the amount in the ones/Units column by the multiplier (the multiplier is the 'bossy' number and says 'I want three groups of four not just one group of four!') and be describing that you now have three groups of four as here we are multiplying 4 by 3:*  *Do the same to the amount in the Tens column and build the product here (30, three times):*  *Move the product of the two Units into the Units answer box:*

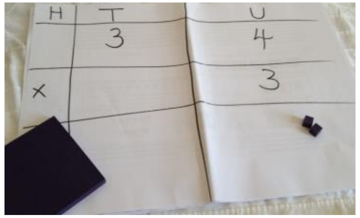


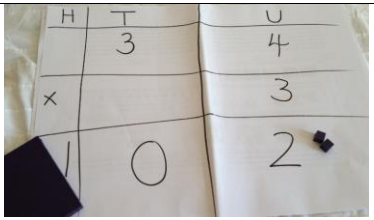
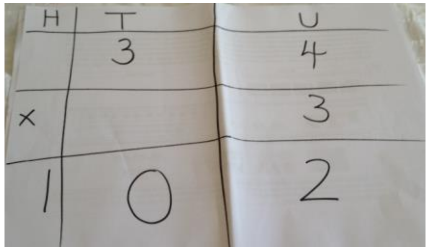
If that product exceeds 9 then it will need to be reorganised in relation to its place value: and then 'carried' over:





Combine the product for the Tens column with the carried amount and consider if it needs to be 'carried' again (i.e. if the total of the carried amount and the product exceeds nine of them):



	 *Then remove the practical apparatus:*  *Children should be able to move to creating this without the use of the concrete resources quite quickly once they have understood conceptually what is happening and how the method works.*